To the Editor:

A recent study of "gulf stream" circulation in bubble columns (Rice and Geary, 1990) uses experimental data described in an early paper of mine (Hills, 1974), but it points out the need to estimate a bubble size since details were not given in my paper.

In fact, the bubble size was not measured, but it can be deduced from measurements given in the 1974 paper, subject to a number of assumptions. Voidage distribution was measured by an impedance probe, and the output from that probe, squared by a trigger circuit, activated two counters, one of which counted the number of 10 kHz pulses passing during the time the probe tip was in air (which led to a point voidage) while the other counted the number of times the probe tip entered a bubble (which led to a point value of bubble frequency). Both voidage and frequency data are plotted in the figures of the 1974 paper.

The ratio of (voidage)/(frequency) clearly represents the average time spent by the probe tip within a bubble at the point of measurement, and it is this quantity that can be manipulated to yield an average bubble size.

Theory

Assume that the bubbles are all spherical, are of uniform radius R, and rise vertically, totally unaffected by the tip of the impedance probe, which thus traces a vertical chord through the bubble. The distribution of these chords horizontally in space should be uniform if the bubbles rise at random, so that a fraction $f(r)dr$ of them lies between r and $(r+dr)$ from the center of the bubble where

$$f(r)dr=\frac{2\pi r\cdot dr}{\pi R^2}=\frac{2r\cdot dr}{R^2}=\frac{d(r^2)}{R^2} \qquad (1)$$

A chord at a distance r from the center of the bubble will have a height c given by:

$$(c/2)^2=R^2-r^2 \qquad (2)$$

so that the distribution of chord lengths is given by:

$$f(c)dc=\frac{d(c/2)^2}{R^2}=\frac{c}{2R^2}dc \qquad (3)$$

The average chord height, $\bar{c}$, is then given by:

$$\bar{c}=\int_0^{2R} cf(c)dc=\int_0^{2R}\frac{c^2dc}{2R^2}=\frac{4}{3}R \qquad (4)$$

If the bubble rise velocity, v_m, is constant at any position, the mean time taken for a bubble to pass the probe tip is given by $\bar{c}/v_m$, so, from the above,

$$\frac{\bar{c}}{v_m}=\frac{4}{3}\frac{R}{v_m}=\frac{\epsilon}{\nu}$$

or

$$v_m=\frac{4}{3}\frac{\nu}{\epsilon}R \qquad (5)$$

Now, averaged over the whole column, v_m is the gas velocity $U_G/\bar{\epsilon}$, so, assuming that R does not vary with radial position, we have:

$$\frac{U_G}{\bar{\epsilon}}=\frac{4}{3}\left(\frac{\nu}{\epsilon}\right)_{\text{ave}}R$$

or

$$R=\frac{3}{4}\frac{U_G}{\bar{\epsilon}}\left(\frac{\epsilon}{\nu}\right)_{\text{ave}} \qquad (6)$$

Thus, by averaging ϵ and the ratio ϵ/ν across the column, we can estimate the mean bubble size.

Results

Table 1 shows the result of these calculations for the seven runs on plate B, which was a sieve plate distributor with 61 uniformly spaced 0.4-mm-diameter holes. The mean bubble diameter lies between 2 and 3 mm at the lowest gas rates, rising to nearly 6 mm at the highest rate; however, at this high rate, considerable coalescence was seen to occur, and the assumptions behind Eq. 6 are unlikely to hold in this region.

Rice and Geary (1990) used an estimation technique to obtain the bubble sizes they needed to apply their theory to the data for plate B, and their estimates are also given in Table 1. For gas rates up to 38 $mm\cdot s^{-1}$ their predictions are some 80% larger than the measured values, and they correspond more closely with the expected size of 4-6 mm in tap water.

Table 1. Calculated Bubble Diameters

Gas rate, $mm\cdot s^{-1}$	5	19	38	64	95	125	169
Bubble dia., mm							
Measured (This Work)	2.35	2.80	4.02	4.14	4.42	5.12	5.83
Estimated (Rice and Geary, 1990)	3.8	5.8	7.6	6.5	5.55	—	—

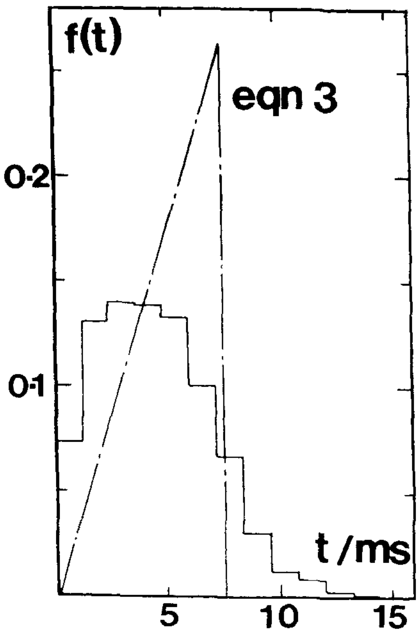

Figure 1. Theoretical distribution function based on Eq. 3.

Many of the assumptions involved in the derivation of Eq. 6 are open to question. Thus, Eq. 2 assumes that the bubbles are spherical, whereas the photographic evidence of many authors is that they are ellipsoids with semiminor axis vertical so that the mean vertical chord height underestimates the equivalent diameter, which could explain the low values in Table 1. Again, it is uncertain how far the impedance probe truly "sees" random vertical chords. Rising bubbles tend to oscillate from side to side, so that not all chords will be vertical; also if a bubble strikes a glancing blow on the probe, it may be deflected so that penetration will not occur and the expected short chord will not be recorded or surface tension may delay separation so that the chord appears too long. Both of these events would affect the expected chord length distribution function, $f(c)$ of Eq. 3.

Some experiments, not reported in the 1974 paper, were performed to measure the distribution of time intervals spent by the impedance probe within a bubble (Hills, 1971). The experiments used an on-line computer to sample the probe output, and a typical example of the distribution is shown in Figure 1. Because of the primitive state of the art at the time, a mechanical relay was used, with a cycle time of 1.2 ms. This proved to be too slow to pick up some of the very short gaps between bubbles, so the computer was counting two bubbles close together as one larger one, overestimating the voidage and underestimating the frequency by some 15% as compared with the countertimers used in the majority of the experiments.

Figure 1 also shows a theoretical distribution function, calculated from Eq. 3 using the same mean time interval (5.7 ms) as the experimental histogram. Comparison of the measured and theoretical distributions yields a number of observations:

- The peak in the measured distribution occurs too soon (at about 4 ms compared with a theoretical $5.7 \times 4/3 = 7.6$ ms).
- There is a significant "tail" of longer times than the peak with a maximum of 15 ms.
- There are too many short time intervals (below 4 ms) and not enough longer ones (4 to 7.6 ms).

The long tail could be due to the presence of large (or slow) bubbles, but it may also be due to the slow response mentioned above which causes the computer to count two bubbles close together as a single large one. However, the slow response cannot explain an overrepresentation of short time intervals, which is most likely to be due to smaller satellite bubbles. It would not need many such smaller bubbles to cause a significant reduction in the mean bubble size, although their contribution to the overall voidage (and hence the driving force for gulf streaming) would be negligible.

Notation

c = length of vertical chord cut by probe
r = radial distance from cebtre of bubble
R = Bubble radius
v_m = rise velocity of bubble
U_G = superficial gas velocity
ϵ = fractional gas holdup, or voidage
ν = bubble frequency at probe tip

Literature cited

Hills, J. H., "Radial Nonuniformity of Velocity and Voidage in a Bubble Column," *Trans. Instn. Chem. Engrs.*, **52**, 1 (1974).

Hills, J. H., "The Effect of Coalescence in Bubble Columns," PhD Thesis, Univ. of Cambridge (1971).

Rice, R. G., and N. W. Geary, "Prediction of Liquid Circulation in Viscous Bubble Columns," *AIChE J.*, **35**, 1339 (1990).

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Reply:

It is interesting and timely that Hills brings forth his earlier data on measured bubble sizes (using the resistance probe method) at the same time as another article of ours (Geary and Rice, 1991) on a new theory to predict this quantity is ready to be published. This new theory, derived with small-holed spargers (such as Hills') in mind, includes corrections for gas (injection) momentum effects and uses an intermediate drag law which was shown to be an improvement over that of Stokes. The new theory compared favorably with wide-ranging literature data. In fact, it is a coincidence that the Figure 10 in this article was generated to predict bubble sizes for Hills' column B, the subject of this correspondence.

The strategy given in the new theory is this: bubble size follows the formation dynamics for increasing gas rate up to the point where it intersects the monotonically decreasing size described by a bubble breakage curve.

Now, Hills' measurements will yield the shortest length scale ($2 \times a$) for bubbles shaped roughly as oblate spheroids, hence volume $V = 4/3\ \pi\ b^2 a$, where b is the major semiaxis length and a is the minor semiaxis distance. Thus, if the major to minor lengths are, as we observe, around 2, then the relationship between Hills' diameter and the equivalent diameter of a sphere (d) is simply:

$$d^* = d/4^{1/3} = 0.63\,d \qquad (1)$$

so that the actual bubble sizes are significantly larger than those computed from resistance probe measurements. We

Table 1. Comparison of New Theory with Data of Hills (Column B)

Superficial Gas Velocity (mm/s)	Hills Raw (d^*) Measured Dia. (mm)	Hills' Data Corrected (Eq. 1) d (mm)	Geary-Rice Formation Size Theory (mm)
5	2.35	3.73	3.6
19	2.80	4.4	4.6
38	4.02	6.38	5.6
64	4.14	6.57	6.0

compare Hills' corrected data with our predictions based on the formation dynamics in Table 1. When account is taken for probe penetration through the minor axis, the predictions from theory compare reasonably well. It is also clear that the turbulence length scale generated by an oblate spheroid will be proportional to the major scale b, and this perhaps explains why the approximate (oversize) bubble size calculated from the Rice-Howell (1987) theory allowed the circulation theory of Rice-Geary (1990) to track Hills' liquid velocity measurements, so closely.

Finally we take note of other errors existing in the resistance probe technique, namely the finite time required for a liquid film to drain from (or reform on) the tip of such finite probes. This causes the output signals to be more rounded; as noted by Clark and coworkers (1990a,b), this leads to ambiguities in the selection of threshold voltages (above this voltage, the signal represents a liquid, and below it, a gas bubble). Moreover, bubbles rising at odd angles will not be pierced at the center and this inaccuracy can only be corrected with a multipoint probe (Steinemann and Buchholz, 1984).

Literature cited

Geary, N. W., and R. G. Rice, "Bubble Size Prediction for Rigid and Flexible Spargers," *AIChE J.*, **37**, p. 000 (1991).

Clark, N. N., J. W. Van Egmond, and E. P. Nebiolo, "The Drift-Flux Model Applied to Bubble Columns and Low Velocity Flows," *Int. J. Multiphase Flow*, **16**, 261 (1990a).

Nebiolo, E. P., and N. N. Clark, "Resistance Probe Measurements of Void Distribution and Circulation Velocity in a Bubble Column," AIChE Meeting, paper 181b, Chicago (1990b).

Rice, R. G., and N. W. Geary, "Prediction of Liquid Circulation in Viscous Bubble Columns," *AIChE J.*, **36**, 1339 (1990).

Rice, R. G., and S. W. Howell, "Bubble Formation from Elastic Holes," *Chem. Eng. Commun.*, **59**, 229 (1987).

Steinemann, J., and R. Buchholz, "Application of an Electrical Conductivity Microprobe for Characterization of Bubble Behavior in Gas-Liquid Bubble Flow," *Part. Charact.*, **1**, 102 (1984).

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